

Lepton polarization correlations in $B \rightarrow K^* \tau^- \tau^+$

S. Rai Choudhury* and Naveen Gaur†

Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India

A. S. Cornell‡

Korea Institute of Advanced Study, Cheongryangri 2-dong, Dongdaemun-gu, Seoul 130-722, Republic of Korea

G. C. Joshi§

School of Physics, University of Melbourne, Victoria 3010, Australia

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In this work we will study the polarizations of both leptons (τ) in the decay channel $B \rightarrow K^* \tau^- \tau^+$. In the case of the dileptonic inclusive decay $B \rightarrow K^* \ell^- \ell^+$, where apart from the polarization asymmetries of a single lepton ℓ , one can also observe the polarization asymmetries of both leptons simultaneously. If this sort of measurement is possible then we can have, apart from decay rate, forward backward asymmetry and the six single lepton polarization asymmetries (three each for ℓ^- and ℓ^+), nine more double polarization asymmetries. This will give us a very useful tool in more strict testing of the standard model (SM) and the physics beyond. We discuss the double polarization asymmetries of the τ leptons in the decay mode $B \rightarrow K^* \tau^- \tau^+$ within the SM and the minimal supersymmetric extension of it.

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I. INTRODUCTION

Flavor changing neutral currents (FCNC) in weak decays provide a fertile ground for testing the structure of weak interactions. Since these decays are forbidden in the tree approximation, they go through higher order loop effects. Consequently they are sensitive to finer details of the basic interactions responsible for the process and as such provide a natural testing ground for any theories beyond the standard model as an example. In the context of B decays, processes involving a dileptonic pair in the final state through the basic quark process $b \rightarrow s \ell^- \ell^+$ provides a wealth of possible experimental data, accessible in the near future, that can be confronted with theoretical predictions. Processes involving this basic quark transition fall into two broad categories, namely the inclusive ones and specific exclusive processes. In both these there have been theoretical investigations involving total cross sections, differential cross sections and polarization studies. The last of these, namely polarization studies of the final state particles is a particularly useful parameter, since the most popular extension of the standard model (SM) predicts considerable modification of their values from SM results [1–5]. Polarizations involving a single lepton have been studied extensively in $B \rightarrow X_s \ell^- \ell^+$ [2,6], $B \rightarrow K^* \ell^- \ell^+$ [7,8,1], $B \rightarrow K \ell^- \ell^+$ [3], $B \rightarrow (\pi, \rho) \ell^- \ell^+$ [4], $B_s \rightarrow \ell^+ \ell^- \gamma$ [5] but recently Bensalam *et al.* [9] have pointed out that the study of simultaneous polarizations of the leptons in the final state provides another observable that can be experimentally measured and provides yet another

parameter in testing models involving physics beyond the standard model. They have, in their, work carried out detailed analysis of the exclusive process $B \rightarrow X_s \ell^- \ell^+$. On similar double polarization asymmetries of both the leptons this process ($B \rightarrow X_s \ell^- \ell^+$) should also get major corrections if we consider extension of SM [10].

In Ref. [9] they have confined themselves to the standard model. But as has been emphasized in many works [11–13] that the supersymmetric extension of the SM gives major corrections to the processes based on the quark level transitions $b \rightarrow s \ell^- \ell^+$. Supersymmetry (SUSY) extends the SM list of terms in the effective Hamiltonian and associated Wilson coefficients; for the quark level process $b \rightarrow s \ell^- \ell^+$ it predicts the presence of two new quark bilinears in the effective Hamiltonian, namely a scalar and a pseudo-scalar one. These new Wilsons come because of the extra neutral Higgs bosons (NHBs) spectrum of SUSY (and two Higgs doublet model) theories [11,12,14]. The effects of these new Wilsons on various kinematical variables such as branching ratios, lepton pair forward backward asymmetries and lepton polarization asymmetries in various inclusive ($B \rightarrow X_s \ell^- \ell^+$, $B \rightarrow X_d \ell^- \ell^+$) [2,11,15] and exclusive ($B \rightarrow K \ell^- \ell^+$, $B \rightarrow K^* \ell^- \ell^+$, $B \rightarrow \ell^- \ell^+ \gamma$, $B \rightarrow \pi \ell^- \ell^+$, $B \rightarrow \rho \ell^- \ell^+$ etc.) [1–3,13,16,17] semi-leptonic and pure leptonic ($B \rightarrow \ell^- \ell^+$) [11] decays of B mesons have been studied in great detail. The new Wilson coefficients (C_{Q_1} and C_{Q_2}) are proportional to $m_\ell m_b \tan^3 \beta$ and hence can be substantial when the lepton is τ and $\tan \beta$ is sufficiently high. We would like to include the effect of NHBs but at the same time focus on an exclusive process $B \rightarrow K^* \ell^- \ell^+$. Experimentally exclusive processes are easier to study but theoretically involve more uncertainties. However for processes such as $B \rightarrow K^* \ell^- \ell^+$ the theoretical uncertainties are somewhat in control since the unknown hadronic matrix element in-

*Electronic address: src@physics.du.ac.in

†Electronic address: naveen@physics.du.ac.in

‡Electronic address: alanc@kias.re.kr

§Electronic address: joshi@tauon.ph.unimelb.edu.au

volved can be related to charged current decay mode of the B meson. The analysis of these has been subject to a lot of theoretical attention and one can use the results there as input to theoretical estimates for the FCNC process. In this paper we take up the study of this exclusive process for determination of all the three polarization parameters, longitudinal, transverse and normal for both the leptons simultaneously. This exclusive process is amongst the more important contribution to the inclusive cross section $B \rightarrow X_s \ell^- \ell^+$ and hopefully will be amongst the first of the processes for which data will become available. Analysis of this process in the SM and in the minimal extension of the standard model have been done by many authors. Lepton polarization asymmetry in $B \rightarrow K^* \ell^- \ell^+$ was first discussed by Geng and Kao [7]. In their later work they also studied SUSY effects in this particular decay mode [8], which as we have already mentioned is important because it is the highest SM branching ratio in all the semi-leptonic decay modes. In particular Aliev *et al.* [1] have given the complete helicity structure of the amplitudes and have focused on asymmetries related to the polarization of the K^* meson. Our study is more in the context of the simultaneous lepton polarization asymmetries and their sensitivities to various input parameters of the MSSM (minimal supersymmetric standard model).

The paper is organized as follows. In the Sec. II we will present the effective Hamiltonian for the process we are considering, and we will write down the matrix element in terms of form factors of the $B \rightarrow K^*$ transition and then will give results of the partial decay rate for $B \rightarrow K^* \ell^- \ell^+$. In Sec. III we will give the analytical results of various polarization asymmetries. The last Sec. IV is devoted to the numerical analysis, discussion and conclusions.

II. EFFECTIVE HAMILTONIAN

The process in which we are interested ($B \rightarrow K^* \ell^- \ell^+$) originates from the quark level transition $b \rightarrow s \ell^- \ell^+$. By integrating out the heavy degrees of freedom from the theory (MSSM here), we get the effective Hamiltonian of the quark level transition $b \rightarrow s \ell^- \ell^+$ [1,3,11,12,14,18]:

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=1}^{10} C_{Q_i}(\mu) Q_i(\mu) \right] \quad (2.1)$$

where O_i are current-current ($i=1,2$), penguin ($i=3, \dots, 6$), magnetic penguin ($i=7,8$) and semi-leptonic ($i=9,10$) operators, and $C_i(\mu)$ are the corresponding Wilson coefficients renormalized at scale μ . They have been given in [19–24]. The additional operators Q_i ($i=1, \dots, 10$), and their Wilson coefficients are due to NHB exchange diagrams and are given in [11,12].

Neglecting the mass of the s quark, the above effective Hamiltonian gives us the following matrix element:

$$\begin{aligned} \mathcal{M} = \frac{\alpha G_F}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left\{ -2 C_7^{eff} \frac{m_b}{q^2} (\bar{s} i \sigma_{\mu\nu} q^\nu P_R b) (\bar{\ell} \gamma^\mu \ell) \right. \\ + C_9^{eff} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell) + C_{10} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell) \\ \left. + C_{Q_1} (\bar{s} P_R b) (\bar{\ell} \ell) + C_{Q_2} (\bar{s} P_R b) (\bar{\ell} \gamma_5 \ell) \right\} \quad (2.2) \end{aligned}$$

where q is the momentum transfer to the lepton pair and is given as $q = p_- + p_+$, where p_- and p_+ are the momenta of ℓ^- and ℓ^+ respectively. $V_{tb} V_{ts}^*$ are the Cabibbo-Kobayashi-Maskawa (CKM) factors and $P_{L,R} = (1 \mp \gamma_5)/2$. In our analysis we will assume that we can factorize $B \rightarrow K^* \ell^- \ell^+$ decay into pure leptonic and hadronic parts.¹

C_9^{eff} has a perturbative part and a part which comes from the long-distance effects due to conversion of the real $c\bar{c}$ into the lepton pair $\ell^- \ell^+$ [6,18,26]:

$$C_9^{eff} = C_9^{per} + C_9^{res} \quad (2.3)$$

where

$$\begin{aligned} C_9^{per} = C_9 + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6) + g(\hat{m}_c, \hat{s}) \\ \times [3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6] - \frac{1}{2} g(1, \hat{s}) \\ \times [4C_3 + 4C_4 + 3C_5 + C_6] - \frac{1}{2} g(0, \hat{s}) [C_3 + 3C_4]. \end{aligned} \quad (2.4)$$

The functions $g(\hat{m}_i, \hat{s})$ arise from the one loop contributions of the four quark operators O_1, \dots, O_6 and have the form

$$\begin{aligned} g(\hat{m}_i, \hat{s}) = -\frac{8}{9} \ln(\hat{m}_i) + \frac{8}{27} + \frac{4}{9} y_i - \frac{2}{9} (2 + y_i) \sqrt{|1 - y_i|} \\ \times \begin{cases} \left[\ln \left(\frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}} \right) - i\pi \right], & 4\hat{m}_i^2 < \hat{s} \\ 2 \arctan \frac{1}{\sqrt{y_i - 1}}, & 4\hat{m}_i^2 > \hat{s} \end{cases} \end{aligned} \quad (2.5)$$

where $y_i = 4\hat{m}_i^2/\hat{s}$. The non-perturbative contribution to C_9^{eff} is associated with the real $c\bar{c}$ resonances in the intermediate states and can be parametrized by using a Breit-Wigner shape, as given in [6,18,26]:

$$\begin{aligned} C_9^{res} = -\frac{3\pi}{\alpha^2} \kappa [3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6] \\ \times \sum_{V=\psi} \frac{\hat{m}_V \text{Br}(V \rightarrow \ell^- \ell^+) \hat{\Gamma}_{total}^V}{\hat{s} - \hat{m}_V^2 + i \hat{m}_V \hat{\Gamma}_{total}^V}. \end{aligned} \quad (2.6)$$

¹There have been attempts in the literature to go beyond a “naive” factorization [25].

The phenomenological parameter κ in the above will be taken to be 2.3 so as to reproduce the correct branching ratio of $\text{Br}(B \rightarrow J/\psi K^* \rightarrow K^* \ell \ell) = \text{Br}(B \rightarrow J/\psi K^*) \text{Br}(J/\psi \rightarrow \ell \ell)$.

Using the definition of the form factors given in Eqs. (A1), (A2), and (A4) we can get the amplitude governing the decay $B \rightarrow K^* \ell^- \ell^+$ as²

$$\begin{aligned} \mathcal{M}^{B \rightarrow K^*} = & \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^* [\{\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} q^\alpha p_K^\beta A - i \epsilon_\mu^* B \\ & + i(p_K)_\mu (\epsilon^* \cdot q) C\} (\bar{\ell} \gamma^\mu \ell) \{\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} q^\alpha p_K^\beta D \\ & - i \epsilon_\mu^* E + i(\epsilon^* \cdot q)(p_K)_\mu F\} (\bar{\ell} \gamma^\mu \gamma_5 \ell) \\ & - iG(\epsilon^* \cdot q)(\bar{\ell} \ell) - iH(\epsilon^* \cdot q)(\bar{\ell} \gamma_5 \ell)] \quad (2.7) \end{aligned}$$

where the coefficients are

$$\begin{aligned} A = & \frac{4\hat{m}_b}{\hat{s}} C_7^{eff} T_1(\hat{s}) + \frac{2A_2(\hat{s})}{1+\hat{m}_{K^*}} C_9^{eff} \\ B = & \frac{2\hat{m}_b}{\hat{s}} (1-\hat{m}_{K^*}) C_7^{eff} T_2(\hat{s}) + A_1(\hat{s})(1+\hat{m}_{K^*}) C_9^{eff} \\ C = & \frac{4\hat{m}_b C_7^{eff}}{\hat{s}} \left[T_2(\hat{s}) + \frac{\hat{s}}{1-\hat{m}_{K^*}} T_3(\hat{s}) \right] + \frac{2A_2(\hat{s})}{1+\hat{m}_{K^*}} C_9^{eff} \\ D = & \frac{2V(\hat{s})}{1+\hat{m}_{K^*}} C_{10} \\ E = & A_1(\hat{s})(1+\hat{m}_{K^*}) C_{10} \\ F = & \frac{2A_2(\hat{s})}{1+\hat{m}_{K^*}} C_{10} \\ G = & \frac{2\hat{m}_{K^*}}{\hat{m}_b} A_0(\hat{s}) C_{Q_1} \\ H = & \frac{2\hat{m}_{K^*}}{\hat{m}_b} A_0(\hat{s}) C_{Q_2} - 2\hat{m}_\ell C_{10} \left(\frac{A_2(\hat{s})}{1+\hat{m}_{K^*}} \right. \\ & \left. + \frac{2\hat{m}_{K^*}}{\hat{s}} [A_3(\hat{s}) - A_0(\hat{s})] \right) \quad (2.8) \end{aligned}$$

where $\hat{s} = s/m_B^2$, $\hat{m}_{K^*} = m_{K^*}/m_B$ and $\hat{m}_\ell = m_\ell/m_B$. From the above expression of the matrix element given in Eq. (2.7) we can get the expression of the dilepton invariant mass spectra as

²In writing this we have used $q_\mu (\bar{\ell} \gamma^\mu \ell) = 0$ and $q_\mu (\bar{\ell} \gamma^\mu \gamma_5 \ell) = 2m_\ell (\bar{\ell} \gamma_5 \ell)$.

$$\frac{d\Gamma(B \rightarrow K^* \ell^+ \ell^-)}{ds} = \frac{G_F^2 \alpha^2 m_B^3}{2^{10} \pi^5} |V_{tb} V_{ts}^*|^2 \lambda^{1/2} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \Delta \quad (2.9)$$

where

$$\begin{aligned} \Delta = & \frac{4}{3} \lambda (\hat{s} + 4\hat{m}_\ell^2) |A|^2 + \frac{2}{3} \frac{(\hat{s} + 2\hat{m}_\ell^2)}{\hat{m}_{K^*}^2 \hat{s}} (\lambda + 12\hat{m}_{K^*}^2 \hat{s}) |B|^2 \\ & + \frac{1}{6} \frac{(\hat{s} + 2\hat{m}_\ell^2)}{\hat{m}_{K^*}^2 \hat{s}} \lambda^2 |C|^2 - \frac{2}{3} \lambda \frac{(1 - \hat{m}_{K^*}^2 - \hat{s})}{\hat{m}_{K^*}^2 \hat{s}} \\ & \times (\hat{s} + 2\hat{m}_\ell^2) \text{Re}(B^* C) + \frac{4}{3} (\hat{s} - 4\hat{m}_\ell^2) \lambda |D|^2 \\ & + \frac{2}{3} \frac{[\lambda(\hat{s} + 2\hat{m}_\ell^2) + 12\hat{m}_{K^*}^2 \hat{s}(\hat{s} - 4\hat{m}_\ell^2)]}{\hat{m}_{K^*}^2 \hat{s}} |E|^2 \\ & + \frac{1}{6} \frac{\lambda}{\hat{m}_{K^*}^2 \hat{s}} [\lambda(\hat{s} + 2\hat{m}_\ell^2) + 24\hat{m}_{K^*}^2 \hat{m}_\ell^2 \hat{s}] |F|^2 \\ & - \frac{2}{3} \lambda \frac{(1 - \hat{m}_{K^*}^2 - \hat{s})}{\hat{m}_{K^*}^2 \hat{s}} (\hat{s} + 2\hat{m}_\ell^2) \text{Re}(E^* F) \\ & + \frac{(\hat{s} - 4\hat{m}_\ell^2)}{\hat{m}_{K^*}^2} \lambda |G|^2 + \frac{\hat{s}}{\hat{m}_{K^*}^2} \hat{s} |H|^2 \\ & + 2 \frac{\hat{m}_\ell}{\hat{m}_{K^*}^2} \lambda [2\text{Re}(E^* H) - (1 - \hat{m}_{K^*}^2 - \hat{s}) \text{Re}(F^* H)] \quad (2.10) \end{aligned}$$

and where $\lambda = \lambda(1, \hat{s}, \hat{m}_{K^*}^2) = 1 + \hat{s}^2 + \hat{m}_{K^*}^4 - 2\hat{s} - 2\hat{m}_{K^*}^2 - 2\hat{s}\hat{m}_{K^*}^2$.

III. LEPTON POLARIZATION ASYMMETRIES

Now we compute the lepton polarization asymmetries of both the leptons defined in the effective four fermion interaction of Eq. (2.2). For this we define the orthogonal vectors S in the rest frame of ℓ^- and W in the rest frame of ℓ^+ , for the polarization of the leptons. L , N and T correspond to the lepton being polarized along the longitudinal, normal and transverse directions respectively [1–3,5,9].

$$\begin{aligned} S_L^\mu & \equiv (0, \mathbf{e}_L) = \left(0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|} \right) \\ S_N^\mu & \equiv (0, \mathbf{e}_N) = \left(0, \frac{\mathbf{p}_{K^*} \times \mathbf{p}_-}{|\mathbf{p}_{K^*} \times \mathbf{p}_-|} \right) \\ S_T^\mu & \equiv (0, \mathbf{e}_T) = (0, \mathbf{e}_N \times \mathbf{e}_L) \quad (3.1) \\ W_L^\mu & \equiv (0, \mathbf{w}_L) = \left(0, \frac{\mathbf{p}_+}{|\mathbf{p}_+|} \right) \end{aligned}$$

$$\begin{aligned}
W_N^\mu &\equiv (0, \mathbf{w}_N) = \left(0, \frac{\mathbf{p}_{K^*} \times \mathbf{p}_+}{|\mathbf{p}_{K^*} \times \mathbf{p}_+|} \right) & S_L^\mu &= \left(\frac{|\mathbf{p}_-|}{m_\ell}, \frac{E_1 \mathbf{p}_-}{m_\ell |\mathbf{p}_-|} \right) \\
W_T^\mu &\equiv (0, \mathbf{w}_T) = (0, \mathbf{w}_N \times \mathbf{w}_L) & W_L^\mu &= \left(\frac{|\mathbf{p}_-|}{m_\ell}, -\frac{E_1 \mathbf{p}_-}{m_\ell |\mathbf{p}_-|} \right).
\end{aligned} \tag{3.2}$$

where \mathbf{p}_+ , \mathbf{p}_- and \mathbf{p}_{K^*} are three momenta of ℓ^+ , ℓ^- and K^* respectively in the c.m. frame of $\ell^-\ell^+$ system. On boosting the above vectors defined by Eqs. (3.1),(3.2) to the c.m. frame of $\ell^-\ell^+$ system, only the longitudinal vector will be boosted while the other two will remain unchanged. The longitudinal vectors after the boost will become

The polarization asymmetries can now be calculated using the spin projector $\frac{1}{2}(1 + \gamma_5 \mathcal{S})$ for ℓ^- and the spin projector $\frac{1}{2}(1 + \gamma_5 \mathcal{W})$ for ℓ^+ .

Equipped with the above we can now define various single lepton and double lepton polarization asymmetries. The single lepton polarization asymmetries are defined as [1-3,5,9]

$$\begin{aligned}
\mathcal{P}_x^- &\equiv \frac{\left(\frac{d\Gamma(S_x, W_x)}{d\hat{s}} + \frac{d\Gamma(S_x, -W_x)}{d\hat{s}} \right) - \left(\frac{d\Gamma(-S_x, W_x)}{d\hat{s}} + \frac{d\Gamma(-S_x, -W_x)}{d\hat{s}} \right)}{\left(\frac{d\Gamma(S_x, W_x)}{d\hat{s}} + \frac{d\Gamma(S_x, -W_x)}{d\hat{s}} \right) + \left(\frac{d\Gamma(-S_x, W_x)}{d\hat{s}} + \frac{d\Gamma(-S_x, -W_x)}{d\hat{s}} \right)}, \\
\mathcal{P}_x^+ &\equiv \frac{\left(\frac{d\Gamma(S_x, W_x)}{d\hat{s}} + \frac{d\Gamma(-S_x, W_x)}{d\hat{s}} \right) - \left(\frac{d\Gamma(S_x, -W_x)}{d\hat{s}} + \frac{d\Gamma(-S_x, -W_x)}{d\hat{s}} \right)}{\left(\frac{d\Gamma(S_x, W_x)}{d\hat{s}} + \frac{d\Gamma(S_x, -W_x)}{d\hat{s}} \right) + \left(\frac{d\Gamma(-S_x, W_x)}{d\hat{s}} + \frac{d\Gamma(-S_x, -W_x)}{d\hat{s}} \right)}
\end{aligned} \tag{3.4}$$

where the subindex x is L , N or T . \mathcal{P}^\pm denotes the polarization asymmetry of the charged lepton ℓ^\pm . Along the same lines we can also define the double spin polarization asymmetries as [9]

$$\mathcal{P}_{xy} \equiv \frac{\left(\frac{d\Gamma(S_x, W_y)}{d\hat{s}} - \frac{d\Gamma(-S_x, W_y)}{d\hat{s}} \right) - \left(\frac{d\Gamma(S_x, -W_y)}{d\hat{s}} - \frac{d\Gamma(-S_x, -W_y)}{d\hat{s}} \right)}{\left(\frac{d\Gamma(S_x, W_y)}{d\hat{s}} + \frac{d\Gamma(-S_x, W_y)}{d\hat{s}} \right) + \left(\frac{d\Gamma(S_x, -W_y)}{d\hat{s}} + \frac{d\Gamma(-S_x, -W_y)}{d\hat{s}} \right)} \tag{3.5}$$

where the subindex x, y are L , N or T .

The expressions of the double polarization asymmetries are

$$\begin{aligned}
\mathcal{P}_{LL} = & \left[\frac{4}{3} \lambda (2\hat{m}_\ell^2 - \hat{s}) |A|^2 + \frac{2}{3} (2\hat{m}_\ell^2 - \hat{s}) (\lambda + 12\hat{m}_{K^*}^2 \hat{s}) |B|^2 + \frac{1}{6} \lambda^2 \frac{(2\hat{m}_\ell^2 - \hat{s})}{\hat{m}_{K^*}^2 \hat{s}} |C|^2 - \frac{2}{3} \lambda \frac{(\hat{s} - 4\hat{m}_\ell^2)}{\hat{m}_{K^*}^2 \hat{s}} (1 - \hat{m}_{K^*}^2 - \hat{s}) \text{Re}(B^* C) \right. \\
& + \frac{4}{3} \lambda (4\hat{m}_\ell^2 - \hat{s}) |D|^2 + \frac{2}{3} \frac{[\lambda (10\hat{m}_\ell^2 - \hat{s}) + 12\hat{s} \hat{m}_{K^*}^2 (4\hat{m}_\ell^2 - \hat{s})]}{\hat{m}_{K^*}^2 \hat{s}} |E|^2 + \frac{1}{6} \frac{\lambda}{\hat{m}_{K^*}^2 \hat{s}} [\lambda (10\hat{m}_\ell^2 - \hat{s}) + 24\hat{s} \hat{m}_{K^*}^2 \hat{m}_\ell^2] |F|^2 \\
& - \frac{2}{3} \frac{\lambda}{\hat{m}_{K^*}^2 \hat{s}} (1 - \hat{m}_{K^*}^2 - \hat{s}) (10\hat{m}_\ell^2 - \hat{s}) \text{Re}(E^* F) + \frac{\lambda}{\hat{m}_{K^*}^2} (\hat{s} - 4\hat{m}_\ell^2) |G|^2 + \frac{\lambda}{\hat{m}_{K^*}^2} \hat{s} |H|^2 \\
& \left. + 4\hat{m}_\ell \frac{\lambda}{\hat{m}_{K^*}^2} [2\text{Re}(E^* H) - (1 - \hat{m}_{K^*}^2 - \hat{s}) \text{Re}(F^* H)] \right] / \Delta
\end{aligned} \tag{3.6}$$

$$\mathcal{P}_{LN} = \frac{1}{2} \frac{\pi \hat{m}_\ell}{\hat{m}_{K^*}^2} \sqrt{\frac{\lambda}{\hat{s}}} \left[\lambda \left\{ \text{Im}(C^* E) - \frac{(\hat{s} - 4\hat{m}_\ell^2)}{2\hat{m}_\ell} \text{Im}(F^* G) + \frac{\hat{s}}{2\hat{m}_\ell} \text{Im}(C^* H) \right\} - (1 - \hat{m}_{K^*}^2 - \hat{s}) \right. \\ \left. \times \left\{ \text{Im}(B^* E) + (1 - \hat{m}_{K^*}^2 - \hat{s}) \text{Im}(B^* F) + \frac{\lambda}{2} \text{Im}(C^* F) + \frac{\hat{s}}{\hat{m}_\ell} \text{Im}(B^* H) - \frac{(\hat{s} - 4\hat{m}_\ell^2)}{\hat{m}_\ell} \text{Im}(E^* G) \right\} \right] / \Delta \quad (3.7)$$

$$\mathcal{P}_{LT} = - \frac{\hat{m}_\ell \pi \sqrt{\lambda(\hat{s} - 4\hat{m}_\ell^2)}}{\hat{m}_{K^*}^2 \hat{s}} \left[(1 - \hat{m}_{K^*}^2 - \hat{s}) \left\{ |E|^2 + \frac{\lambda}{4} |F|^2 + \frac{\hat{s}}{2\hat{m}_\ell} [\text{Re}(E^* H) - \text{Re}(B^* G)] \right\} - (\lambda + 2\hat{m}_{K^*}^2 \hat{s}) \text{Re}(E^* F) \right. \\ \left. - 2\hat{s} \hat{m}_{K^*}^2 \{ \text{Re}(B^* D) + \text{Re}(A^* E) \} + \frac{\hat{s}}{4\hat{m}_\ell} \lambda \{ \text{Re}(C^* G) - \text{Re}(F^* H) \} \right] / \Delta \quad (3.8)$$

$$\mathcal{P}_{NL} = -\mathcal{P}_{LN} \quad (3.9)$$

$$\mathcal{P}_{NN} = \frac{2}{3} \frac{\lambda}{\hat{m}_{K^*}^2} \left[\hat{m}_{K^*}^2 (\hat{s} - 4\hat{m}_\ell^2) (|A|^2 - |D|^2) - \left\{ \left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) + 24 \frac{\hat{m}_{K^*}^2 \hat{m}_\ell^2}{\lambda} \right\} \left(|B|^2 - \frac{1}{4} |F|^2 \right) \right. \\ \left. + \left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) \left\{ |E|^2 - \frac{\lambda}{4} |C|^2 \right\} + (1 - \hat{m}_{K^*}^2 - \hat{s} h) \left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) \{ \text{Re}(B^* C) - \text{Re}(E^* F) \} \right. \\ \left. - \frac{3}{2} (\hat{s} - 4\hat{m}_\ell^2) |G|^2 + \frac{3}{2} \hat{s} |H|^2 + 6\hat{m}_\ell \{ 2\text{Re}(E^* H) - (1 - \hat{m}_{K^*}^2 - \hat{s}) \text{Re}(F^* H) \} \right] / \Delta \quad (3.10)$$

$$\mathcal{P}_{NT} = \frac{2}{3} \frac{\lambda}{\hat{m}_{K^*}^2} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \left[2\hat{s} \hat{m}_{K^*}^2 \text{Im}(A^* D) - (1 - \hat{m}_{K^*}^2 - \hat{s}) \{ \text{Im}(F^* B) + \text{Im}(E^* C) + 3\hat{m}_\ell \text{Im}(G^* F) \} \right. \\ \left. + \frac{\lambda}{2} \text{Im}(F^* C) + 2\text{Im}(E^* B) + 6\hat{m}_\ell \text{Im}(G^* E) - 3\hat{s} \text{Im}(G^* H) \right] / \Delta \quad (3.11)$$

$$\mathcal{P}_{TL} = - \frac{\pi \hat{m}_\ell}{\hat{m}_{K^*}^2 \hat{s}} \sqrt{\lambda(\hat{s} - 4\hat{m}_\ell^2)} \left[(1 - \hat{m}_{K^*}^2 - \hat{s}) \left\{ |E|^2 + \frac{1}{4} |F|^2 + \frac{\hat{s}}{2\hat{m}_\ell} [\text{Re}(B^* G) + \text{Re}(E^* H)] \right\} \right. \\ \left. - \lambda \text{Re}(E^* F) + 2\hat{m}_{K^*}^2 \hat{s} \{ \text{Re}(B^* D) + \text{Re}(A^* E) \} - \hat{s} \lambda \left\{ \text{Re}(G^* C) - \frac{1}{2} \text{Re}(H^* F) \right\} \right] / \Delta \quad (3.12)$$

$$\mathcal{P}_{TN} = -\mathcal{P}_{NT} \quad (3.13)$$

$$\mathcal{P}_{TT} = - \frac{2}{3} \frac{\lambda}{\hat{s} \hat{m}_{K^*}^2} \left[\hat{m}_{K^*}^2 \hat{s} \{ (\hat{s} + 4\hat{m}_\ell^2) |A|^2 - (\hat{s} - 4\hat{m}_\ell^2) |D|^2 \} - \{ \lambda (\hat{s} - 2\hat{m}_\ell^2) - 24\hat{s} \hat{m}_\ell^2 \hat{m}_{K^*}^2 \} |B|^2 - \frac{1}{4} \lambda (\hat{s} - 4\hat{m}_\ell^2) |C|^2 \right. \\ \left. + (1 - \hat{m}_{K^*}^2 - \hat{s}) (\hat{s} - 2\hat{m}_\ell^2) \text{Re}(C^* B) - (10\hat{m}_\ell^2 - \hat{s}) |E|^2 - \frac{1}{4} \{ (10\hat{m}_\ell^2 - \hat{s}) \lambda - 24\hat{s} \hat{m}_\ell^2 \hat{m}_{K^*}^2 \} |F|^2 + (1 - \hat{m}_{K^*}^2 - \hat{s}) \right. \\ \left. \times (10\hat{m}_\ell^2 - \hat{s}) \text{Re}(E^* F) + \frac{3}{2} \hat{s} (\hat{s} - 4\hat{m}_\ell^2) |G|^2 + \frac{3}{2} |H|^2 + 3\hat{m}_\ell \hat{s} \{ 2\text{Re}(E^* H) - (1 - \hat{m}_{K^*}^2 - \hat{s} h) \text{Re}(H^* F) \} \right] / \Delta \quad (3.14)$$

where Δ is given in Eq. (2.10).

From their definitions, Eqs. (3.1)–(3.5), polarization asymmetries relating the longitudinal (L) and transverse (T) spin orientations are parity odd whereas the normal one (N)

is parity even. Consequently of the various double polarization asymmetries, Eqs. (3.6)–(3.14), only \mathcal{P}_{LN} and \mathcal{P}_{TN} are parity odd. However, the basic weak interaction Hamiltonian is not invariant under parity transformation so that from par-

TABLE I. Our standard model predictions of the averaged value of the observables.

$\text{BR}(B \rightarrow K^* \tau^- \tau^+)$	\mathcal{P}_{LL}	\mathcal{P}_{LN}	\mathcal{P}_{LT}	\mathcal{P}_{NL}	\mathcal{P}_{NN}	\mathcal{P}_{NT}	\mathcal{P}_{TL}	\mathcal{P}_{TN}	\mathcal{P}_{TT}
1.29×10^{-7}	-0.299	-0.09	-0.329	0.09	-0.036	-0.0016	-0.037	0.0016	-0.011

ity symmetry considerations alone, no conclusion can be drawn about the vanishing or otherwise of these asymmetries.

Since we are dealing with local Lorentz invariant theories, time reversal invariance is synonymous with CP invariance. In the decay process $B^0 \rightarrow K^* \ell^+ \ell^-$, neither the initial nor the final state is an eigenstate of CP so that CP invariance or otherwise of the theory relate amplitudes of this process with its conjugate process $\bar{B} \rightarrow \bar{K}^* \ell^+ \ell^-$. It should be noted that there are terms in our matrix element which involve a triple product and thus naively have the appearance of a T-odd interaction. This is not correct since we are dealing with an effective Hamiltonian which includes the effect of strong phases which gives fake CP -violation signals even when the basic Hamiltonians are all CP conserving.

For the charge conjugate process the corresponding amplitudes will have their CKM factor conjugated. For $b \rightarrow s$ type of transition such as the one considered here, the CKM phase becomes an overall phase factor since we can neglect the very small $b \rightarrow u$ couplings. Possible CP violating phases in the CKM factor thus will not show up in any decay rate. Other possible sources of CP violation, for example, can come from the supersymmetry breaking parameter μ becoming complex. The present calculation however takes all supersymmetric breaking soft terms in the Lagrangian to be real so that we have effective CP -invariance of our results. The implications of these for possible measurements of double polarization asymmetries are remarked upon at the end of Sec. IV.

IV. NUMERICAL ANALYSIS, RESULTS AND DISCUSSION

We have performed the numerical analysis of all the kinematical variables which we have presented in Sec. III. The parameters which we have used in our numerical analysis are listed in Appendix B. We have quoted our averaged standard model values of all these variables in Table I.

We have also analyzed the effects of supersymmetry on the observables. For the numerical analysis we have considered MSSM, this is the simplest of the SUSY models with the least number of parameters. One of the major parameters of MSSM is $\tan \beta$ which is the ratio of the vev (vacuum expectation value) of the two Higgs doublets of MSSM. We will focus on the MSSM parameter space at large $\tan \beta$. The reason for this being that in the large $\tan \beta$ region of MSSM parameter space the contributions of NHB exchange becomes very important for quark level semi-leptonic transitions $b \rightarrow s \ell^- \ell^+$ especially when final state lepton is either a muon μ or tau τ . This point has been noted in many FCNC semi-leptonic [12,13] and pure dileptonic transitions [11,14]. Actually if we consider MSSM then we have to extend the set of SM Wilson coefficients, for semi-leptonic transitions

we have to introduce two new Wilsons, namely, C_{Q_1} and C_{Q_2} . These coefficients come from the exchange of NHBs and are proportional to $m_b m_\ell \tan^3 \beta / m_h$, where m_ℓ , m_b and m_h are lepton, b- quark and Higgs boson mass respectively. So as we can see that if lepton is either μ or τ and the Higgs mass is suitable then the new Wilsons (C_{Q_1} and C_{Q_2}) can have fairly large values. The values of C_{Q_1} and C_{Q_2} also depend on other MSSM parameters such as chargino masses and splittings, stop masses and splittings etc. But as is well known these masses and splittings are constrained by the process $B \rightarrow X_s \gamma$ [27]. In our numerical analysis we will take a 95% C.L. bound [28]:

$$2 \times 10^{-4} < \text{BR}(B \rightarrow X_s \gamma) < 4.5 \times 10^{-4} \quad (4.1)$$

which is in agreement with CLEO and ALEPH results.

We shall now discuss the models used in our numerical analysis. The MSSM is defined on the basis of four basic assumptions (for a review of the MSSM refer to [29]): (i) Minimal gauge group, which is $SU(3)_c \times SU(2)_L \times U(1)_Y$ which is the SM group also, (ii) minimal particle content, (iii) R-parity conservation, (iv) minimal set of soft SUSY breaking terms. If we use only these conditions then the model which is constructed is called the unconstrained MSSM (also called the phenomenological MSSM as one can readily study the phenomenology of it). But this sort of model gives rise to many phenomenological problems such as FCNC, unusually large CP violation, incorrect value of Z mass etc. But these sorts of problems can be resolved once we make some assumptions such as all SUSY breaking parameters are real and hence no new source of CP violation, matrices for sfermion masses and trilinear couplings are diagonal which prevents tree level FCNC processes, first and second generation sfermion universality which helps us in getting away with the K^0 - \bar{K}^0 mixing problem.

But there is another way of solving all the problems of the unconstrained MSSM model, which is to require all the soft SUSY breaking parameters have a universal value at some GUT (grand unified theory) scale. If we make the universal values of these parameters real then even the CP violation problem is solved. This is the case in case of constrained MSSM and minimal supergravity (mSUGRA) models.

Aside from the universality of all the gauge coupling constants in mSUGRA models the other conditions are the following: universality of all the scalar masses, unification of all the gaugino masses and universality of all the trilinear couplings at the GUT scale. With all these constraints if we impose the condition of correct electroweak symmetry

breaking then we have another parameter which is $\text{sgn}(\mu)$ ³ and $\tan\beta$ which is the ratio of the vev of the Higgs doublets. So in all the mSUGRA frameworks we have five parameters

$$m, \quad M, \quad A, \quad \tan\beta, \quad \text{sgn}(\mu).$$

But it is interesting to study the departure of these sorts of models. By departure we mean what would happen if we relax some of the above mentioned conditions of mSUGRA model. With this sort of relaxing of conditions we effectively introduce additional parameters in the model. One can study such relaxed models also and have reasonable predictions of such SUSY models if the number of new parameters introduced is not large.⁴ There can be many options available; such as relaxation of universality of gaugino masses at GUT, relaxation of universality of scalar masses at GUT etc.

In our analysis we will choose to relax the condition of universality of the scalar masses at GUT. We will assume non-universality of sfermionic and Higgs masses, i.e. the sfermions and Higgs have different universal masses at GUT scale. This sort of model we will call the rSUGRA model. With this sort of relaxation we have to introduce another parameter, this parameter we will consider to be the mass of pseudo-scalar Higgs boson m_A .

We shall now discuss the constraints put on the parameters of our models. We will consider only that region of parameter space which satisfies the $B \rightarrow X_s \gamma$ constraints given in Eq. (4.1). Within the SM this decay is mediated by loops containing the charge 2/3 quarks and W bosons. For the set of parameters given in Appendix B our SM value of $\text{BR}(B \rightarrow X_s \gamma)$ turns out to be 3.4×10^{-4} . In SUSY theories there are additional contributions to $b \rightarrow s \gamma$ which come from the chargino-stop loop, top quark and charged Higgs loop and loops involving gluino and neutralinos.⁵ Also this branching ratio constrains only the magnitude of C_7^{eff} . For $\text{sgn}(\mu) > 0$ the chargino-stop contribution interferes destructively with SM and charged Higgs contribution.⁶ The chargino stop contributions grows with $\tan\beta$ and because of its destructive interference with the SM and charged Higgs contributions can give us a region of allowed parameter space. Recently there have been calculations about the next to leading order (NLO) QCD corrections to the $b \rightarrow s \gamma$ decay rate in SUSY [31] but for our work we will use the LO calculations as far as the SUSY corrections are concerned [27,30].

As has been emphasized in many works [2,32] the universality of scalar masses is not a constraint in SUGRA. To suppress large $K^0 - \bar{K}^0$ mixing, the requirement is that all squarks should have universal mass at GUT scale. So that one can relax the condition of universality of scalar masses at GUT scale. This sort of model we have called rSUGRA. The

³ μ is the SUSY Higgs mass parameter.

⁴Effectively this sort of model lies somewhere in between the unconstrained MSSM and the mSUGRA model.

⁵The contribution due to the loops involving gluino and neutralinos are small as shown in [27,30].

⁶In our sign convention for μ it appears in the chargino mass matrix with a positive sign.

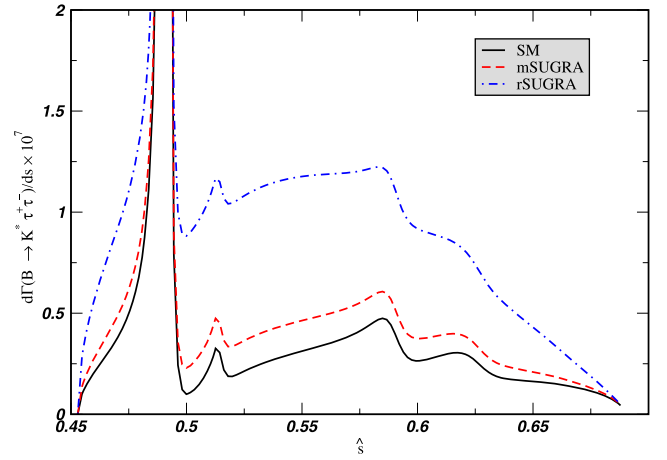


FIG. 1. Branching ratio of $B \rightarrow K^* \tau^- \tau^+$ variation with scaled invariant mass of dileptons. Parameters of mSUGRA are $m = 200$ GeV, $M = 600$ GeV, $A = 0$, $\tan\beta = 45$ and $\text{sgn}(\mu)$ being positive. The additional parameter in rSUGRA model (the mass of pseudo-scalar Higgs boson) is taken to be $m_A = 270$ GeV.

advantage of this model arises as here we can have some handle on the Higgs boson mass and as has been emphasized earlier in many works the new Wilson coefficients C_{Q_1} and C_{Q_2} are very sensitive to Higgs masses. So in this sort of model one can more easily see the dependence of various observables on the new Wilson coefficients.

We also present the results of the average polarization asymmetries. The averaging is defined as

$$\langle \mathcal{P} \rangle \equiv \frac{\int_{(m_B - m_{K^*})^2/m_B^2}^{m_B^2/m_B^2} \mathcal{P} \frac{d\Gamma}{d\hat{s}} d\hat{s}}{\int_{(m_B - m_{K^*})^2/m_B^2}^{m_B^2/m_B^2} \frac{d\Gamma}{d\hat{s}} d\hat{s}}. \quad (4.2)$$

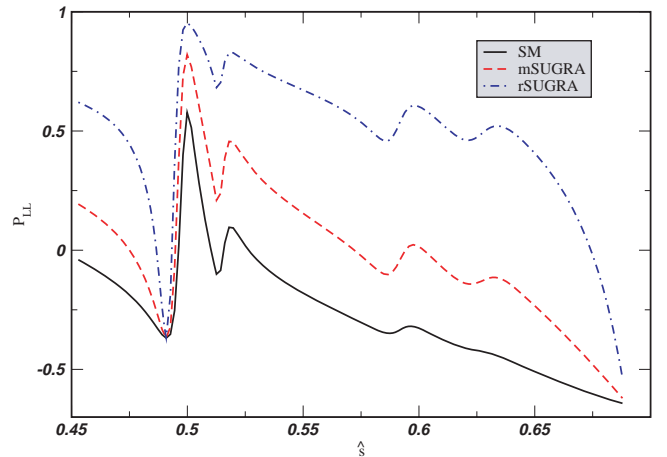


FIG. 2. \mathcal{P}_{LL} variation with scaled invariant mass of dileptons. Parameters of mSUGRA are $m = 200$ GeV, $M = 600$ GeV, $A = 0$, $\tan\beta = 45$ and $\text{sgn}(\mu)$ being positive. The additional parameter in rSUGRA model (the mass of pseudo-scalar Higgs boson) is taken to be $m_A = 270$ GeV.

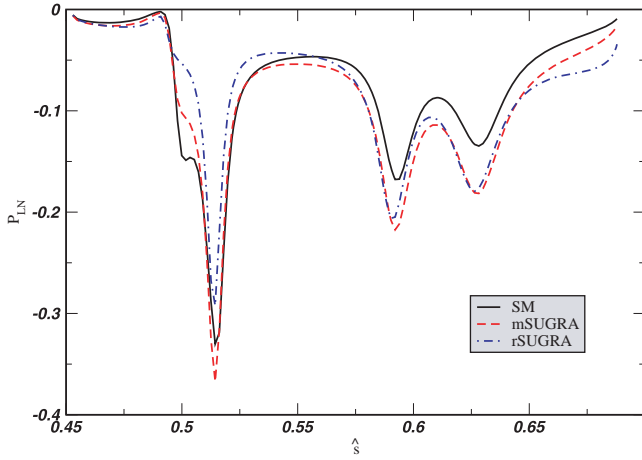


FIG. 3. \mathcal{P}_{LN} variation with scaled invariant mass of dileptons. Parameters of mSUGRA are $m=200$ GeV, $M=600$ GeV, $A=0$, $\tan\beta=45$ and $\text{sgn}(\mu)$ being positive. The additional parameter in rSUGRA model (the mass of pseudo-scalar Higgs boson) is taken to be $m_A=270$ GeV.

Although we have given the expected values of all the double polarization asymmetries with the SM in Table I, but in the graphs we have shown only those polarization asymmetries whose integrated values exceeds 0.1 either in the SM or in the various SUGRA models we have considered.

In Fig. 1 we have plotted the variation of differential decay rate with the scaled invariant mass of the dileptons. In Figs. 2–7 we have plotted the various double polarization asymmetries. In Fig. 8 we have shown the variation of the branching ratio of $B \rightarrow K^* \tau^- \tau^+$ as a function of the pseudo-scalar Higgs mass in the rSUGRA model. In Fig. 9 we have shown the variation of branching ratio as a function of $\tan\beta$ in the mSUGRA model. Similarly in Figs. 10–15 we have shown the variation of the various integrated double polarization asymmetries as a function of the mass of the pseudo-

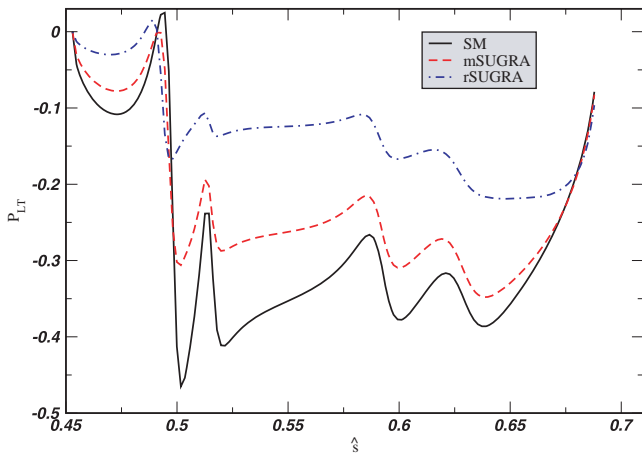


FIG. 4. \mathcal{P}_{LT} variation with scaled invariant mass of dileptons. Parameters of mSUGRA are $m=200$ GeV, $M=600$ GeV, $A=0$, $\tan\beta=45$ and $\text{sgn}(\mu)$ being positive. The additional parameter in rSUGRA model (the mass of pseudo-scalar Higgs boson) is taken to be $m_A=270$ GeV.

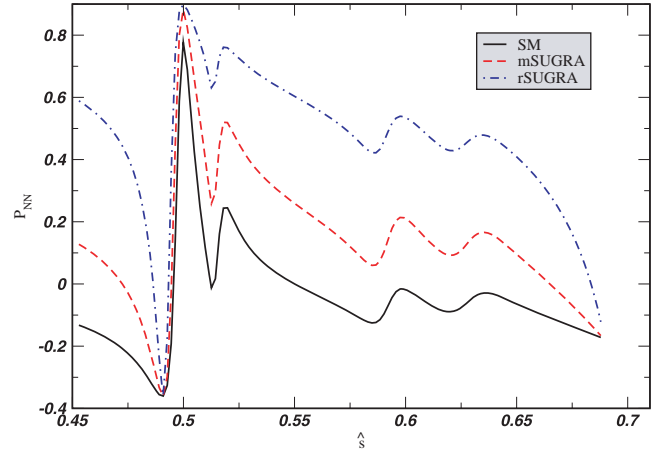


FIG. 5. \mathcal{P}_{NN} variation with scaled invariant mass of dileptons. Parameters of mSUGRA are $m=200$ GeV, $M=600$ GeV, $A=0$, $\tan\beta=45$ and $\text{sgn}(\mu)$ being positive. The additional parameter in rSUGRA model (the mass of pseudo-scalar Higgs boson) is taken to be $m_A=270$ GeV.

scalar Higgs boson mass m_A in the rSUGRA model for various values of $\tan\beta$. In Figs. 16–21 we have shown the variation of various integrated double polarization asymmetries as a function of $\tan\beta$ in the mSUGRA model for various values of m (the unified mass of sleptons and squarks at GUT scale).

It is clear from the figures that several of these polarization asymmetries are sizable and that they are sensitive to the inclusion of the supersymmetric contributions both with regards to the magnitude and sometimes with regard to the sign also. The SM predictions are quite definitive; the only parameter not yet totally fixed is the mass m_b , however, varying this within the acceptable limits does not change the values of the various asymmetries appreciably. Experimental observations of these polarization asymmetries will provide useful confirmatory verification of the validity of MSSM in

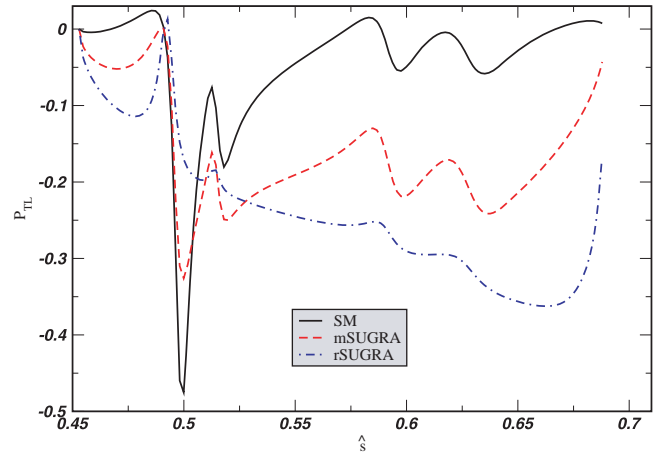


FIG. 6. \mathcal{P}_{TL} variation with scaled invariant mass of dileptons. Parameters of mSUGRA are $m=200$ GeV, $M=600$ GeV, $A=0$, $\tan\beta=45$ and $\text{sgn}(\mu)$ being positive. The additional parameter in rSUGRA model (the mass of pseudo-scalar Higgs boson) is taken to be $m_A=270$ GeV.

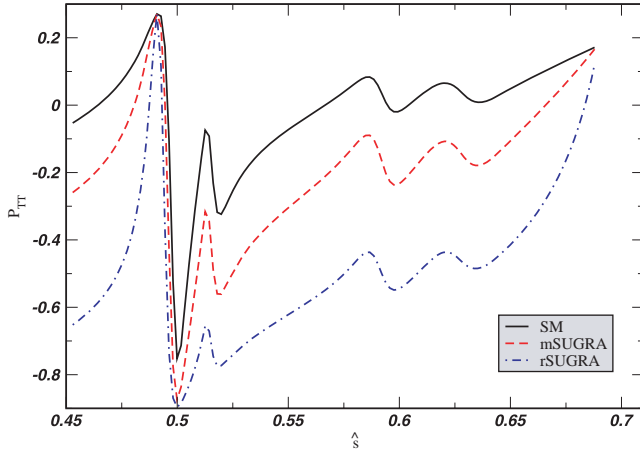


FIG. 7. \mathcal{P}_{TT} variation with scaled invariant mass of dileptons. Parameters of mSUGRA are $m=200$ GeV, $M=600$ GeV, $A=0$, $\tan\beta=45$ and $\text{sgn}(\mu)$ being positive. The additional parameter in rSUGRA model (the mass of pseudo-scalar Higgs boson) is taken to be $m_A=270$ GeV.

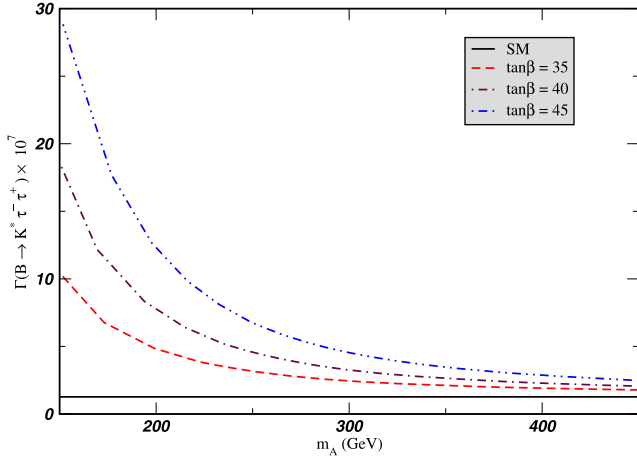


FIG. 8. Total branching ratio of $B \rightarrow K^* \tau^- \tau^+$ variation with m_A (in GeV) for various values of $\tan\beta$ in rSUGRA model. Other model parameters are $m=200$ GeV, $M=450$ GeV, $A=0$.

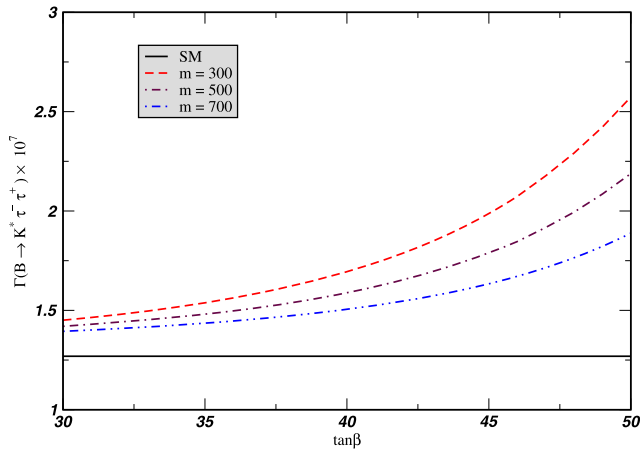


FIG. 9. Total branching ratio of $B \rightarrow K^* \tau^- \tau^+$ variation with $\tan\beta$ for various sets of m in mSUGRA model. Other model parameters are $M=500$ GeV, $A=0$.

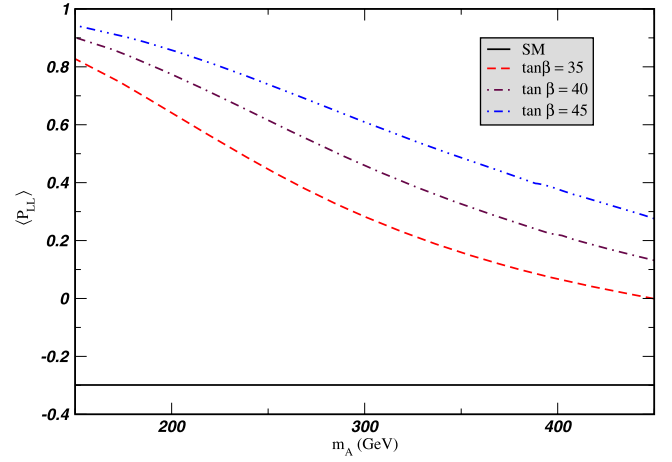


FIG. 10. $\langle \mathcal{P}_{LL} \rangle$ variation with m_A (in GeV) for various values of $\tan\beta$ in rSUGRA model. Other model parameters are $m=200$ GeV, $M=450$ GeV, $A=0$.

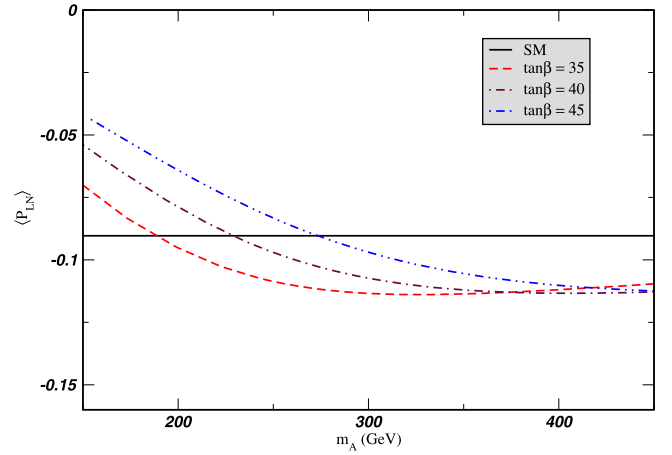


FIG. 11. $\langle \mathcal{P}_{LN} \rangle$ variation with m_A (in GeV) for various values of $\tan\beta$ in rSUGRA model. Other model parameters are $m=200$ GeV, $M=450$ GeV, $A=0$.

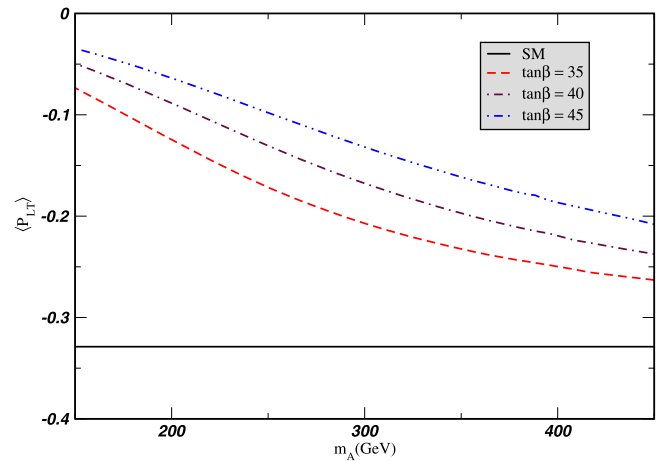


FIG. 12. $\langle \mathcal{P}_{LT} \rangle$ variation with m_A (in GeV) for various values of $\tan\beta$ in rSUGRA model. Other model parameters are $m=200$ GeV, $M=450$ GeV, $A=0$.

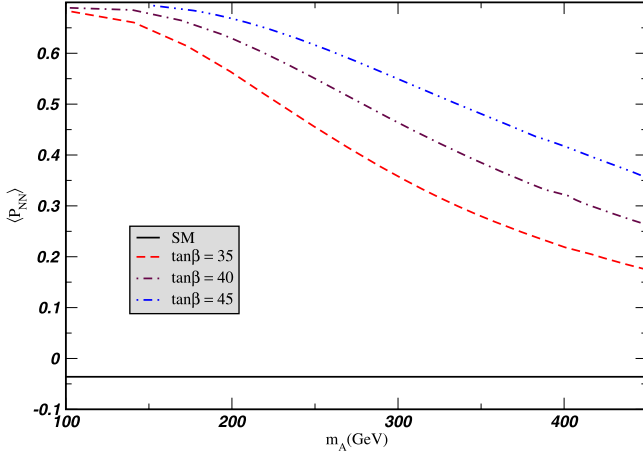


FIG. 13. $\langle \mathcal{P}_{NN} \rangle$ variation with m_A (in GeV) for various values of $\tan\beta$ in rSUGRA model. Other model parameters are $m = 200$ GeV, $M = 450$ GeV, $A = 0$.

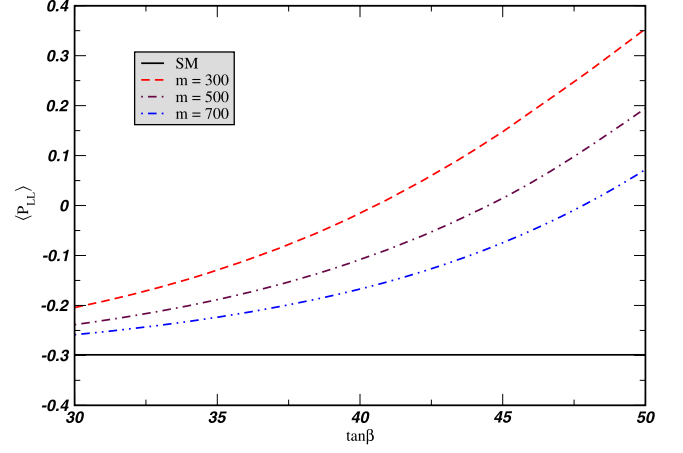


FIG. 16. $\langle \mathcal{P}_{LL} \rangle$ variation with $\tan\beta$ for various sets of m in mSUGRA model. Other model parameters are $M = 500$ GeV, $A = 0$.

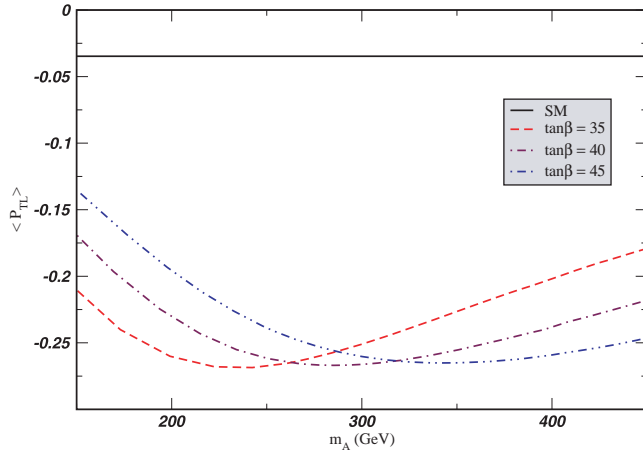


FIG. 14. $\langle \mathcal{P}_{TL} \rangle$ variation with m_A (in GeV) for various values of $\tan\beta$ in rSUGRA model. Other model parameters are $m = 200$ GeV, $M = 450$ GeV, $A = 0$.

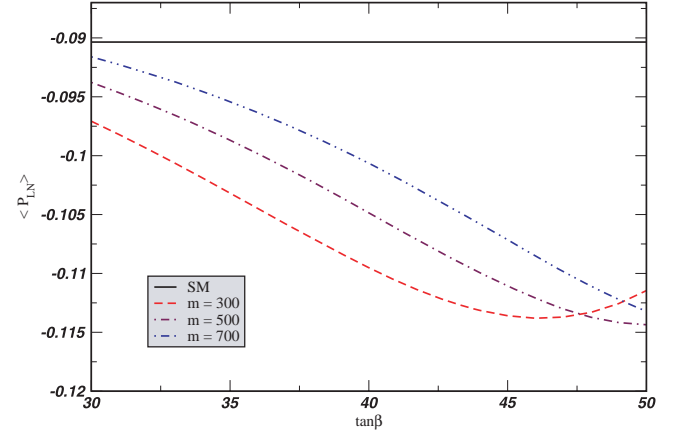


FIG. 17. $\langle \mathcal{P}_{LN} \rangle$ variation with $\tan\beta$ for various sets of m in mSUGRA model. Other model parameters are $M = 500$ GeV, $A = 0$.

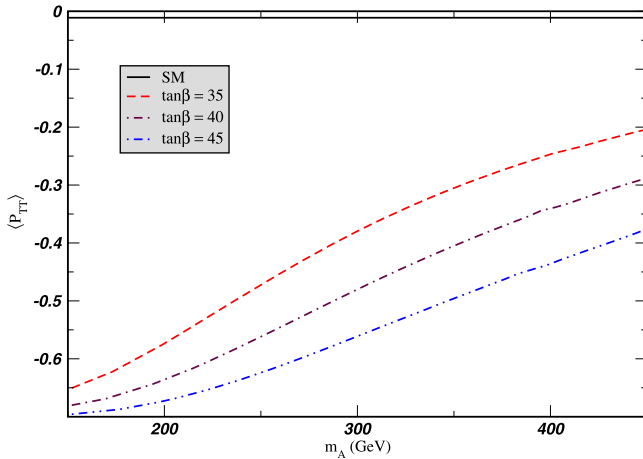


FIG. 15. $\langle \mathcal{P}_{TT} \rangle$ variation with m_A (in GeV) for various values of $\tan\beta$ in rSUGRA model. Other model parameters are $m = 200$ GeV, $M = 450$ GeV, $A = 0$.

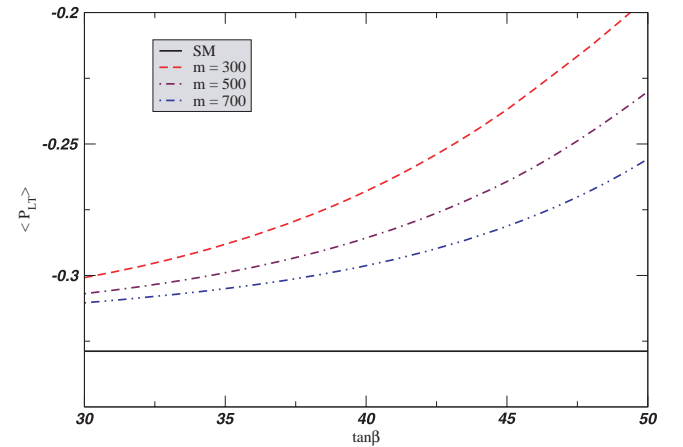


FIG. 18. $\langle \mathcal{P}_{LT} \rangle$ variation with $\tan\beta$ for various sets of m in mSUGRA model. Other model parameters are $M = 500$ GeV, $A = 0$.

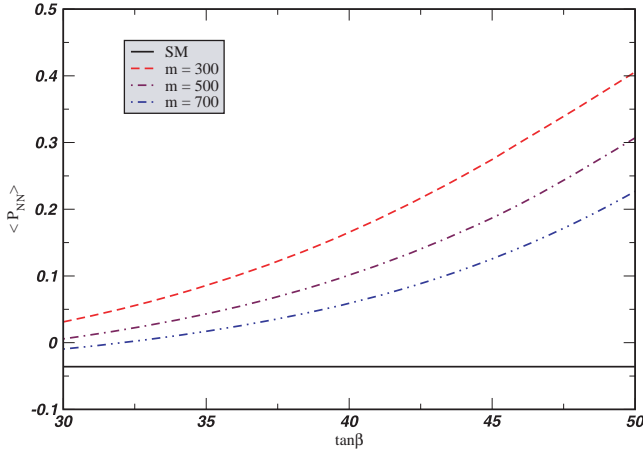


FIG. 19. $\langle \mathcal{P}_{NN} \rangle$ variation with $\tan \beta$ for various sets of m in mSUGRA model. Other model parameters are $M=500$ GeV, $A=0$.

rare decays of the B meson together with other experimental signatures such as single lepton polarization, forward-backward asymmetry, etc.

In presenting our results we have omitted showing the values of the polarization asymmetry parameters \mathcal{P}_{NT} and \mathcal{P}_{TN} , since their values are less than 0.01 and thus would be nearly impossible for observation with or without SUSY contributions. However, if future experiments arise with values for these which are much larger than that, it will be a clear indication of physics not only beyond the SM but also beyond the MSSM within the range of parameters allowed by other experimental constraints.

Finally, our results pertain to the decay $B \rightarrow K^*(p_k) \ell^+(p_+) \ell^-(p_-)$. As discussed in the last section, for the charge conjugate process with the momenta unchanged, i.e. $\bar{B} \rightarrow \bar{K}^*(p_k) \ell^-(p_+) \ell^+(p_-)$ the polarization asymmetries ($\bar{\mathcal{P}}_{ij}$) will be given by $\pm \mathcal{P}_{ji}$, with the negative sign for \mathcal{P}_{LN} and \mathcal{P}_{NT} and the positive sign for the others. Observations of these asymmetries for B and \bar{B} decays would obviously need tagging of the B mesons. Observations with-

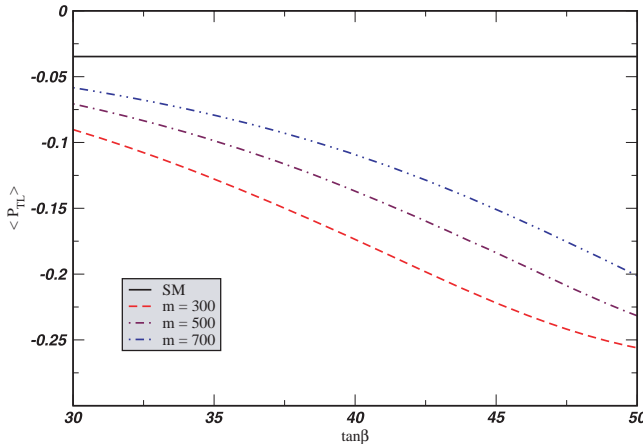


FIG. 20. $\langle \mathcal{P}_{TL} \rangle$ variation with $\tan \beta$ for various sets of m in mSUGRA model. Other model parameters are $M=500$ GeV, $A=0$.

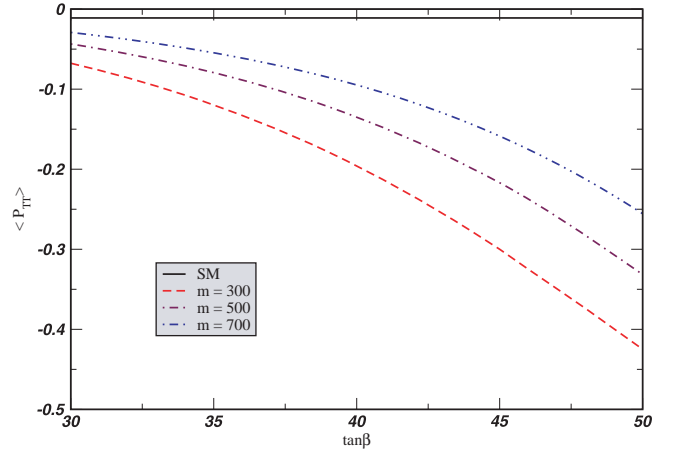


FIG. 21. $\langle \mathcal{P}_{TT} \rangle$ variation with $\tan \beta$ for various sets of m in mSUGRA model. Other model parameters are $M=500$ GeV, $A=0$.

out tagging with an equal number of B and \bar{B} mesons would clearly produce a null value for \mathcal{P}_{LN} and \mathcal{P}_{NT} but would yield value of \mathcal{P}_{LL} , \mathcal{P}_{NN} , \mathcal{P}_{TT} and $(\mathcal{P}_{LT} + \mathcal{P}_{TL})$. The situation will change in the CKM-suppressed related process $B \rightarrow \rho l^+ l^-$ where because of the presence of two terms in the effective Hamiltonian with different CKM factors, the CKM phase would show up in the interference term and would change sign in going from this process to its conjugate one. Observations of asymmetries in such a process with mixtures of B and \bar{B} , as and when they become experimentally accessible, would provide another way of studying the CP violation through CKM phases.

ACKNOWLEDGMENTS

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APPENDIX A: FORM FACTORS

The exclusive decay $B \rightarrow K^* \ell^- \ell^+$ can be described in terms of matrix elements of the quark operators in Eq. (2.2) over meson states, which can be parametrized in terms of form factors. For $B \rightarrow K^* \ell^- \ell^+$ the matrix elements in terms of form factors of the $B \rightarrow K^*$ transition are [33,18]

$$\begin{aligned}
 & \langle K^*(p_K) | (V-A)_\mu | B(p_B) \rangle \\
 &= -i \epsilon_\mu^* (m_B + m_{K^*}) A_1(s) + i (p_B + p_K)_\mu (\epsilon^* \cdot p_B) \\
 & \times \frac{A_2(s)}{m_B + m_{K^*}} + i q_\mu (\epsilon^* \cdot p_B) \frac{2m_{K^*}}{s} [A_3(s) - A_0(s)] \\
 & + \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_B^\alpha p_K^\beta \frac{2V(s)}{m_B + m_{K^*}}
 \end{aligned} \tag{A1}$$

TABLE II. Form factors for $B \rightarrow K^*$ transition.

	F(0)	c_1	c_2
$A_1(s)$	0.337	0.602	0.258
$A_2(s)$	0.282	1.172	0.567
$A_0(s)$	0.471	1.505	0.710
$V(s)$	0.457	1.482	1.015
$T_1(s)$	0.379	1.519	1.030
$T_2(s)$	0.399	0.517	0.426
$T_3(s)$	0.260	1.129	1.128

and

$$\begin{aligned}
& \langle K^*(p_K) | \bar{s} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B(p_B) \rangle \\
&= i \epsilon_{\mu\nu\alpha\beta} \epsilon_\nu^* p_B^\alpha p_K^\beta 2T_1(s) + T_2(s) \{ \epsilon_\mu^* (m_B^2 - m_K^2) \\
&\quad - (\epsilon^* \cdot p_B)(p_B + p_K)_\mu \} \times T_3(s) (\epsilon^* \cdot p_B) \\
&\quad \times \left\{ q_\mu - \frac{s}{m_B^2 - m_K^2} (p_B + p_K)_\mu \right\} \quad (A2)
\end{aligned}$$

where in the above equations p_K and ϵ_μ^* are the four-momentum and polarization vector of the K^* meson respectively. By using the equations of motion we can get a relationship between the form factors as

$$A_3(s) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(s) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(s). \quad (A3)$$

The matrix element of the scalar and pseudo-scalar currents is arrived at by multiplying Eq. (A1) by q^μ on both the sides:

$$\langle K^*(p_K) | \bar{s} (1 \pm \gamma_5) b | B(p_B) \rangle = -2i \frac{m_{K^*}}{m_b} (\epsilon^* \cdot q) A_0(s). \quad (A4)$$

For the form factors we use the results given in [33] where we parametrize the form factors as

$$F(\hat{s}) = F(0) \exp(c_1 \hat{s} + c_2 \hat{s}^2). \quad (A5)$$

The related parameters (c_1 and c_2) are given in Table II.

APPENDIX B: INPUT PARAMETERS

$$m_B = 5.26 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV},$$

$$m_\mu = 0.106 \text{ GeV}, \quad m_\tau = 1.77 \text{ GeV},$$

$$m_w = 80.4 \text{ GeV}, \quad m_z = 91.19 \text{ GeV},$$

$$V_{tb} V_{ts}^* = 0.0385, \quad \alpha = \frac{1}{129}, \quad m_{K^*} = 0.892 \text{ GeV},$$

$$\Gamma_B = 4.22 \times 10^{-13} \text{ GeV},$$

$$G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}.$$

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